Astrometric Detection of Double Gravitational Microlensing Events

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ABSTRACT

If a gravitational microlensing event is caused by a widely separated binary lens and the source approaches both lens components, the source flux is successively magnified by the individual lenses: double microlensing events. If events are observed astrometrically, double lensing events are expected to occur with an increased frequency due to the long range astrometric effect of the companion. We find that although the trajectory of the source star image centroid shifts of an astrometric double lensing event has a distorted shape from both of the elliptical ones induced by the individual single lens components, event duplication can be readily identified by the characteristic loop in the trajectory formed during the source's passage close to the companion. We determine and compare the probabilities of detecting double lensing events from both photometric and astrometric lensing observations by deriving analytic expressions for the relations between binary lensing parameters to become double lensing events. From this determination, we find that for a given set of the binary separation and the mass ratio the astrometric probability is roughly an order higher than the photometric probability. Therefore, we predict that a significant fraction of events that will be followed up by using future high precision interferometeric instruments will be identified as double lensing events.

Subject headings: gravitational lensing – binaries: general

1. Introduction

One of the most important characteristics of microlensing light curve is that it does not repeat. However, if an event is caused by a widely separated binary lens and the source approaches both lens components, the source flux is successively magnified by the individual lenses: double microlensing events (Di Stefano & Mao 1996). Due to the special geometric condition, however, the chance to become a double lensing event is rare. Therefore, double lensing events have been neglected in the previous and current microlensing searches (Udalski et al. 2000; Alcock et al. 2000; Derue et al. 2001; Bond et al. 2001).

Although lensing events have until now been observed only photometrically, they can also be observed astrometrically by the use of high precision interferometric instruments that will be available in the near future, such as those to be mounted on space-based platforms, e.g. the *Space Interferometry Mission* (SIM) and the *Global Astrometric Interferometer for Astrophysics* (GAIA), and those to be mounted on 10m class ground-based tele-

scopes, e.g. Keck and VLT. If an event is astrometrically observed by using these instruments, one can measure the displacement of the source star image centroid position with respect to its unlensed position (centroid shifts, $\boldsymbol{\delta}$). Once the trajectory of $\boldsymbol{\delta}$ is measured, the lens mass can be better constrained (Miyamoto & Yoshii 1995; Hog, Novikov & Polanarev 1995; Walker 1995; Paczyński 1998; Boden, Shao & Van Buren 1998; Han & Chang 1999). Recently, astrometric microlensing observation is accepted as one of the SIM long term projects (A. Gould, private communication), and thus astrometric followup observations of events detected from the ground-based photometric surveys will become a routine process.

One important characteristic of astrometric lensing behavior is that the astrometric effect endures to a large lens-source separation where the photometric effect is negligible (Miralda-Escudé 1996). Then, it is expected that the chance for the source trajectory to enter the astrometrically effective lensing region of the companion will be larger, and thus the probability of detecting astrometric double lensing events[1] will also be larger. In this paper, we investigate the general properties of astrometric double lensing events and estimate and compare the probabilities of detecting double lensing events from both photometric and astrometric lensing observations.

The paper is organized as follows. In § 2, we investigate the general properties of double lensing events by presenting and comparing the centroid shift trajectories and the light curves of events caused by an example wide separation binary. In § 3, we derive analytic expressions for the relations between binary lensing parameters to become photometric and astrometric double lensing events and estimate the detection probabilities by using these relations. In § 4, we summarize new findings and conclude.

2. Properties of Double Lensing Events

The region of photometrically effective lensing region is usually expressed in terms of the Einstein ring radius (Vietri & Ostriker 1983; Turner, Ostriker & Gott 1984), which is related to the lens parameters by

$$\theta_{\rm E} = \sqrt{\frac{4Gm}{c^2}} \left(\frac{1}{D_{\rm ol}} - \frac{1}{D_{\rm os}} \right)^{1/2}, \qquad (1)$$

where m is the lens mass and $D_{\rm ol}$ and $D_{\rm os}$ are the distances to the lens and the source, respectively. If an event is caused by a binary where the projected separation between the lens components is significantly larger than the sum of the Einstein ring radii of the individual lens components, the resulting light curve is approximated by the superposition of those of the events where the individual lens components work as independent single lenses, i.e.

$$A \sim A_1 + A_2 - 1. \qquad (2)$$

Here A_i represents the magnification of each single lens event and the subscripts $i = 1$ and 2 are used to denote the quantities involved with the primary[2] and the companion, respectively. Each single lens event light curve is related to the lensing parameters by

$$A_i = \frac{u_i^2 + 2}{u_i \sqrt{u_i^2 + 4}}; \qquad \boldsymbol{u}_i = \left(\frac{t - t_{0,i}}{t_{{\rm E},i}} \right) \hat{\boldsymbol{x}} \ + \ \beta_i \ \hat{\boldsymbol{y}}, \quad (3)$$

where $\boldsymbol{u}_i$ represents the separation vector between the source and the lens normalized by the Einstein ring radius of the related lens, $\theta_{{\rm E},i}$, $t_{{\rm E},i}$ is the time required for the source to cross $\theta_{{\rm E},i}$ (Einstein time scales), $t_{0,i}$ is the time of the source's closest approach to each lens, and β_i is the lens-source separation (normalized by $\theta_{{\rm E},i}$) at that moment (impact parameter). The units vectors $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are parallel with and normal to the lens-source transverse motion. By entering the Einstein ring (i.e. $u_i \leq 1.0$), the source flux is magnified by $A_i \geq 3/\sqrt{5} \sim 1.34$.

Similar to the light curve, the centroid shift of the wide binary event is approximated by the superposition of the centroid shift vectors induced by the individual single lenses (An & Han 2001), i.e.,

$$\boldsymbol{\delta} \sim \boldsymbol{\delta}_1 + \boldsymbol{\delta}_2, \qquad (4)$$

where the individual centroid shift vectors are represented by

$$\boldsymbol{\delta}_i = \frac{\boldsymbol{u}_i}{u_i^2 + 2} \theta_{{\rm E},i}. \qquad (5)$$

For a single lens event, the centroid shift follows an elliptical trajectory (astrometric ellipse), whose shape (eccentricity) is determined by the impact parameter and the size (semi-major axis) is directly proportional to the angular Einstein ring radius (Walker 1995; Jeong, Han & Park 1999).

To show the properties of astrometric double lensing events, in Figure 1, we present several example centroid shift trajectories of events caused by a widely separated binary lens system. To compare with the photometric lensing behavior, we also present the light curves for the corresponding events. In each panel, we present three different curves, where the solid curve is that of the exact binary lens event, the dotted curve is that obtained

[1] We define the "astrometric double lensing event" as "the event where the path of the source star passes through both of the astrometrically effective lensing regions of the individual lens components".

[2] Here the primary denotes the binary component that causes earlier magnification of the source flux and the companion is reserved for the other component causing later magnification.

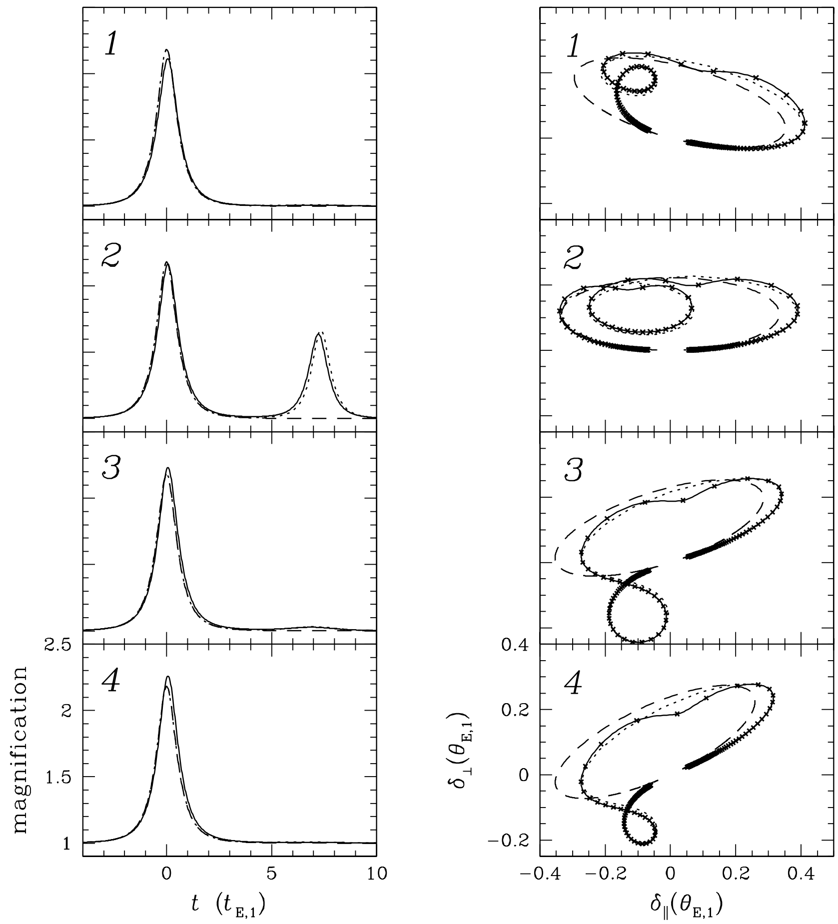

Fig. 1.— Example light curves and centroid shift trajectories of events caused by a widely separated binary lens system. Each panel contains three curves, where the solid curve is that of the exact binary lens event, the dotted curve is that obtained by the single lensing superposition, and the dashed curve is that expected without the presence of the companion. The lens system geometry responsible for each event is presented in Fig. 2. The number in each panel corresponds to the source trajectory number also marked in Fig. 2. Time and length are expressed in units of $t_{\rm E,1}$ and $\theta_{\rm E,1}$, i.e. the Einstein time scale and the Einstein ring radius of the lens causing earlier magnification of the source flux. $\delta_\parallel$ and $\delta_\perp$ represent the components of $\boldsymbol{\delta}$ that are parallel with and normal to the binary axis.

by the single lensing superposition, and the dashed curve is that expected without the presence of the companion. The lens system has a mass ratio between the components of $q = m_2/m_1 = 0.5$ and they are separated by $d = 7.35$ in units of $\theta_{\rm E,1}$. The source trajectory responsible for each event is marked in Figure 2. To show the astrometrically effective region of the companion, we also present the excess centroid shift vectors of the binary lens system from the centroid shifts induced by the primary lens alone, i.e. $\Delta\boldsymbol{\delta} = \boldsymbol{\delta} - \boldsymbol{\delta}_1$. Greyscales are used to show the regions where the amount of excess is greater than $\Delta\delta = 5\%$, 10%, 15% and 20% of $\theta_{\rm E,1}$, respectively. We note that time and lengths are expressed in units of $t_{\rm E,1}$ and $\theta_{\rm E,1}$ because the event involved with the primary lens will be the standard of all measurements. To show the changing rate of the centroid position, we mark the centroid positions ('x' symbol) measured with a time interval of $t_{\rm E,1}/4$. For an event with $t_{\rm E,1}/4$ is a month, therefore, this time interval corresponds to a week.

From Fig. 1 and 2, one finds the following three important characteristics of astrometric double lensing events.

1. First, unlike the light curve which is composed of two well separated light curves induced by the individual lens components, the centroid shift trajectory is not simply composed of two separated astrometric ellipses, but has a distorted shape from both of the elliptical trajectories. This is because the centroid shift results from the vector sum of those induced by the individual lenses, while the

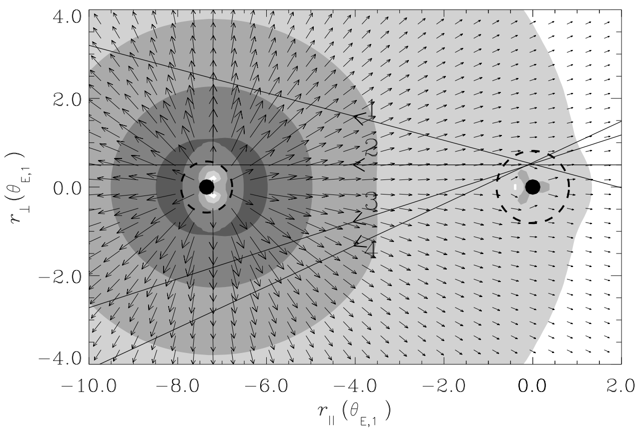

Fig. 2.— The lens system geometry responsible for the events whose light curves and centroid shift trajectories are presented in Fig. 1. The lens system has a mass ratio between the components of $q = 0.5$ and they are separated by $d = 7.35$ in units of $\theta_{E,1}$. The straight lines with arrows represent the source trajectories. The lenses are marked by filled dots and the primary is located at the origin and the companion is to the left. The dashed circles represent the Einstein rings of the individual lenses. The vectors represent the excess centroid shifts $\Delta\boldsymbol{\delta}$ and the grey-scales are used to show the regions where the excesses are greater than $\Delta\delta$ = 5%, 10%, 15% and 20% of $\theta_{E,1}$, respectively.

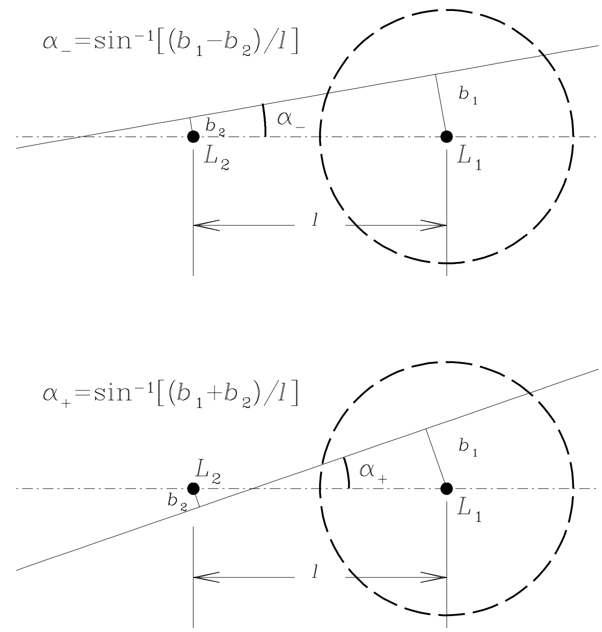

Fig. 3.— Geometry of double lensing events. The binary lens components are marked by filled dots and the primary is to the right. The values b_1 and b_2 represent the smallest separations between the source trajectory to the individual lenses, which are separated by ℓ. The angle $\alpha_{\pm}$ is the orientation angle of the source trajectory with respect to the binary axis. The upper panel is for the case when the closest points on the source trajectory from the individual lenses are on the same side of the binary axis and the lower panel is when the points are located on the opposite sides.

magnification results from the scalar sum of the individual magnifications.

2. Second, despite the distorted shape of the centroid shift trajectory, event duplication can be readily identified by the characteristic loop in the trajectory of $\boldsymbol{\delta}$ formed during the source's passage close to the companion.

3. Third, due to the long range astrometric effect of the companion, as expected, astrometric detection of the event duplication will be possible even when the separation between the source trajectory and the companion is considerable. By contrast, photometric detection of a double lensing event will be possible only when the source approaches very close to the companion.

3. Probabilities of Double Lensing Events

In the previous section, we have investigated the general properties of astrometric double lensing events. In this section, we derive analytic expressions for the relations between the binary lensing parameters to become photometric and astrometric double lensing events. By using these relations, we then determine and compare the probabilities of detecting double lensing events from both photometric and astrometric lensing observations.

3.1. Photometric Double Lensing Events

If b_1 and b_2 represent the smallest separations from the source trajectory to the individual lens components, where projected separation between them is $\ell \gg \theta_{E,1} + \theta_{E,2}$, the orientation angle of the source trajectory with respect to the binary axis is represented by

$$\alpha_{\pm} = \sin^{-1}\left(\frac{b_1 \pm b_2}{\ell}\right) \tag{6}$$

(Di Stefano & Scalzo 1999). Here the sign "−" is for the case when the the closest points on the source trajectory from the individual lenses are located on the same side of the binary axis and the "+" sign is when the points are located on the opposite sides (see Figure 3). When all lengths are normalized in units of $\theta_{E,1}$, equation (6)

is expressed by

$$\alpha_{\pm} = \sin^{-1}\left(\frac{\beta_1 \pm \sqrt{q}\beta_2}{d}\right), \tag{7}$$

where $d = \ell/\theta_{\rm E,1}$. Let us define $\beta_{\rm th}$ as the threshold impact parameter to the companion such that among events for which the light variation induced by the primary is detected, only a fraction of events whose source trajectories approaching the companion closer than $\beta_{\rm th}$ will become double lensing events. Then, under the definition of a photometric double lensing event as "the event where the source trajectory enters both of the Einstein ring radii of the individual binary lens components", the condition to become a photometric double lensing event is $\beta_2 \leq \beta_{\rm th} = 1.0$, and thus the detection probability is computed by

$$P_{\rm ph} = \frac{\alpha_{\rm ph,+} - \alpha_{\rm ph,-}}{\pi}, \tag{8}$$

where

$$\alpha_{\rm ph,\pm} = \sin^{-1}\left(\frac{\beta_1 \pm \sqrt{q}}{d}\right). \tag{9}$$

3.2. Astrometric Double Lensing Events

Astrometrically, lensing has a longer range effect than the photometric effect, and thus a new definition of the threshold impact parameter is required. Let us define $\delta_{\rm th}$ as the threshold amount of the centroid shift induced by the companion that is required for an event to be identified as an astrometric double lensing event. Then, the threshold impact parameter corresponding to $\delta_{\rm th}$ is obtained by solving equation (5) with respect to the lens-source separation, resulting in

$$\beta_{\rm th} = \frac{1}{2}\left(\frac{1}{\delta_{\rm th}/\theta_{\rm E,2}} + \sqrt{\frac{1}{(\delta_{\rm th}/\theta_{\rm E,2})^2} - 8}\right). \tag{10}$$

Note that equation (5) is a quadratic equation of u and thus solving the equation results in two values of $\beta_{\rm th}$. The two solutions correspond respectively to the lens-source separations that are smaller and larger than $u = \sqrt{2}$, at which δ_2 becomes maximum. Since we are interested only in the maximum allowed lens-source separation for the event to be identified as a double lensing event, we take the larger value. If we define a detectable astrometric double lensing event as "an event with centroid shift induced by the companion larger than f fraction of the Einstein ring radius of the primary lens", i.e. $\delta_{\rm th}/\theta_{\rm E,2} = f\theta_{\rm E,1}/\theta_{\rm E,2} = f/\sqrt{q}$, equation (10) is expressed in terms of f by

$$\beta_{\rm th} = \frac{1}{2}\left(\frac{\sqrt{q}}{f} + \sqrt{\frac{q}{f^2} - 8}\right). \tag{11}$$

Then the probability of detecting astrometric double lensing event is represented by

$$P_{\rm ast} = \frac{\alpha_{\rm ast,+} - \alpha_{\rm ast,-}}{\pi}, \tag{12}$$

where

$$\begin{aligned} \alpha_{\rm ast,+} &= \sin^{-1}\left(\frac{\beta_1 + \sqrt{q}\beta_{\rm th}}{d}\right), \\ \alpha_{\rm ast,-} &= \sin^{-1}\left(\frac{\left|\beta_1 - \sqrt{q}\beta_{\rm th}\right|}{d}\right), \end{aligned} \tag{13}$$

for $d > \beta_1 + \sqrt{q}\beta_{\rm th}$,

$$\alpha_{\rm ast,+} = \frac{\pi}{2}, \qquad \alpha_{\rm ast,-} = \sin^{-1}\left(\frac{\left|\beta_1 - \sqrt{q}\beta_{\rm th}\right|}{d}\right), \tag{14}$$

for $\left|\beta_1 - \sqrt{q}\beta_{\rm th}\right| < d \leq \beta_1 + \sqrt{q}\beta_{\rm th}$, and

$$\alpha_{\rm ast,\pm} = \pm\frac{\pi}{2}, \tag{15}$$

for $d \leq \left|\beta_1 - \sqrt{q}\beta_{\rm th}\right|$. We note that since the astrometrically effective lensing region of the companion can be large enough to extend to or even beyond the position of the primary, the threshold orientation angles $\alpha_{\rm ast,\pm}$ have different sets of values depending on the relative size of the astrometrically effective region of the companion to the binary separation.

In Figure 4, we present the probabilities of detecting astrometric and photometric double lensing events as contour maps in the parameter space of the binary separation (in units of $\theta_{\rm E,1}$) and the mass ratio, (d, q). For the computation $P_{\rm ast}$, we set $f = 0.1$. If the mass and the location of the primary lens are $m = 0.3\ M_\odot$ and $D_{\rm ol} = 4$ kpc, the Einstein ring radius is $\theta_{\rm E,1} \sim 500$ μ-arcsec, and thus the imposed threshold centroid shift induced by the companion corresponds to $\delta_{\rm th} \sim 50$ μ-arcsec. We note that the astrometric precision of the SIM will be as low as several μ-arcsec, and thus this amount of shift can be easily detected. Since astrometric followup will be performed only for events where the source flux variation is identified by the photometric surveys, we compute the probabilities by setting the range of the impact parameter to the primary lens to be $0 < \beta_1 \leq 1.0$, and the presented probabilities are the mean values. Note that since the lens causing later magnification (companion) can be heavier than the lens causing earlier magnification (primary), the mass ratio can be larger than $q = 1.0$. To locate the unlensed source position, which is the reference position of $\boldsymbol{\delta}$ measurements, astrometric followup observations of each event will be performed for a long period of time, and thus event duplication can be identified for events caused by a considerably wide separation binary. For an event with $t_{\rm E,1} \sim 20$ days, the time

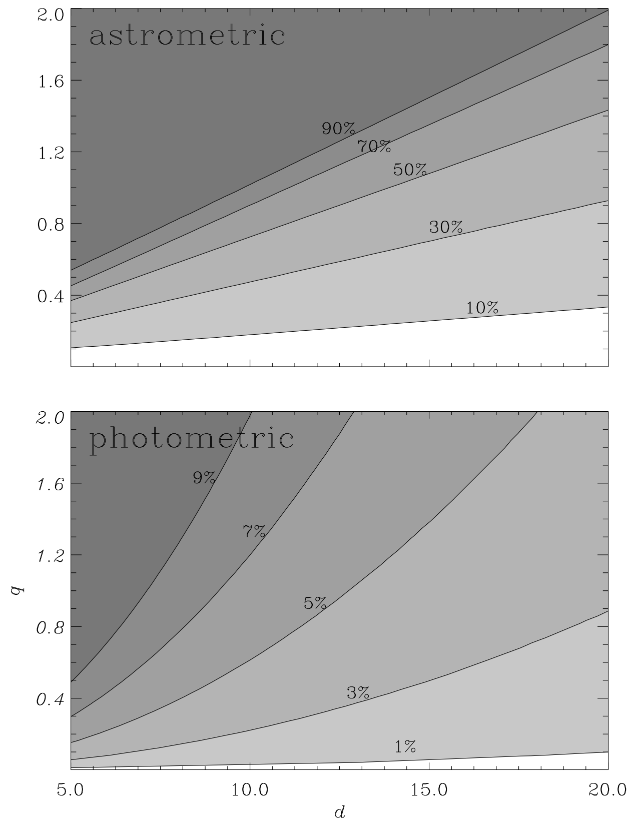

Fig. 4.— Comparison of the probabilities of detecting double lensing events from astrometric and photometric lensing observations. Note that since the lens causing later magnification (companion) can be heavier than the lens causing earlier magnification (primary), the mass ratio can be larger than $q = 1.0$.

required to identify the duplication caused by a companion with $d \sim 20$ will be slightly more than a year. From the figure, one finds that for a given set of d and q, the astrometric probability is roughly an order higher than the photometric probability.

4. Conclusion

We have investigated the properties of double lensing events expected to be identified from future astrometric lensing observations. From this investigation, we find that although the centroid shift trajectory of an astrometric double lensing event has a distorted shape from those of the elliptical ones induced by the individual lens components, the event duplication can be readily identified from the characteristic loop formed during the source's approach close to the companion. We have also determined and compared the probabilities of detecting double lensing events from both photometric and astrometric lensing observations by deriving analytic expressions for the relations between the binary lensing parameters to become double lensing events. From this determination, we find that for a given set of the binary separation and the mass ratio, the probability to detect astrometric double lensing events is roughly an order high than the probability to detect photometric double lensing events. Therefore, we predict that a significant fraction of events that will be followed up by using future high precision interferometeric instruments will be identified as double lensing events.

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